Non-linearity in unemployment and demand-side policy for Australia, Japan and the USA

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1. Introduction

This paper extends Mitchell (2001c) and Mitchell and Muysken (2002b) by exploring more fully the non-linearities in unemployment and GDP growth rates in Australia, Japan and the United States. Mitchell (2001c) presented diagrammatic evidence for unemployment and vacancy rates in Australia, Japan and the U.S., which reveals considerable non-linearity in their time series behaviour. Mitchell and Muysken (2002b) use the Current Depth of Recession (CDR) approach, introduced by Beaudry and Koop (1993), to test one aspect of this non-linearity – the asymmetry in the response of the unemployment rate to negative and positive shocks, specifically, that negative shocks impact more strongly than positive shocks. They find that the CDR model helps to explain the evolution of unemployment in Australia and the Netherlands with unemployment rising sharply in negative cycles and falling more slowly in the upturns.

The issue of non-linearity is important to policy makers aiming to minimise the costs of economic fluctuations. Mitchell (1927: 290) long ago noted that “the most violent declines exceed the most considerable advances.” He thus recognised that macroeconomic models should accommodate asymmetric behaviour. Yet standard macroeconomic models of time series behaviour (which underpin the NAIRU concept) generally employ smooth functions with some allowance for persistence. While Keynesian economics was attacked for lacking micro-foundations, the foundations generally proposed “are typically such that adjustment cost functions are convex and differentiable – an invariably quadratic. This implies that the response to shocks is partial and continuous” (Holly and Stannett, 1995: 767). Further, cyclical non-linearity and asymmetry means that the stochastic time series will vary over the cycle, which means that “the summary statistics of the stochastic behaviour of any variable (e.g. mean growth rates, measures of persistence, impulse response functions, conditional variance, etc.) all need to be conditioned on the state of the business cycle” (Acemoglu and Scott, 1994: 1303).

The dominant macroeconomic policy paradigm is firmly entrenched on using monetary policy to control inflation accompanied by supportive (restrictive) fiscal policy. If non-linearities are ignored then the costs of maintaining restrictive economic conditions are likely to be higher and endure for longer than otherwise.
Mitchell and Carlson (2001) argue that this dominant policy practice has led policy makers to deliberately and persistently deflate their economies under the false impression that they have to ensure the economy is operating at the natural rate of unemployment. They term this the NAIRU approach. Most OECD economies have failed to generate sufficient jobs over the last 25 years to match the growth in their labour forces.

Section 2 introduces phase diagrams for Australia, Japan and the US unemployment, vacancy and real GDP growth rates to analyse the presence of attractors and cyclical ellipses. Employment and labour force growth rates are also analysed for Australia. The analysis supported by the diagrams suggests that considerable non-linearity exists in several of the time series, particularly the unemployment and vacancy series for Australia. In turn this suggests that negative demand shock are extremely costly. Section 3 introduces a conceptual framework for testing for cyclical asymmetry (deepness and sharpness) and finds strong support for the hypothesis that both forms of asymmetry are present in unemployment rates for Australia, Japan and the US. Section 4 extends the CDR analysis in Mitchell and Muysken (2002b) to Japan and the US. The results reveal that there is statistically significant CDR non-linearity in all the unemployment rates for Australia, Japan and the US. Section 5 introduces the smooth transition autoregressive (STAR) class of non-linear models. The estimation for Australia shows that complex non-linear dynamics are present in the unemployment rate and that severe non-linear behaviour is associated with cyclical downturns. The switch to recession is rapid with the unemployment rate rising sharply and falling more slowly. Section 6 outlines the policy implications of the paper and concludes that demand side policy must be used to avoid these sharp excursions into non-linearity.

2. **Attractors and cyclical ellipses**

Phase diagrams for the unemployment rate and the vacancy rate, respectively are shown in Figures 1 and 2. The current values of the respective time series are plotted on the y-axis against the lagged value of the same series on the x-axis. The years noted refer to the current year’s unemployment rate. These diagrams convey four sources of information about the time series behaviour of the variables. First, they provide information on whether cycles are present in the data. Second, the presence of “attractor points” or “centres of gravity” can be determined. The points might loosely
be construed as the “centre of the ellipses traced out in such a plot” (Ormerod, 1994: 154). Third, the magnitude of the cycles can be inferred by the size of the cyclical ellipses around the attractor points. Fourth, the persistence (strength) of the attractor point can be determined by examining the extent to which it disciplines the cyclical observations following a shock. Weak attractors will not dominate a shock and the relationship will shift until a new attractor point asserts itself.

Significant differences between the three countries are revealed in Figure 1. Australia shifted its attractor in the 1974-76 period and the two subsequent recessions have oscillated around this higher point with varying cyclical magnitude. The explanation for Australia’s persistently high unemployment rate revolves around the factors that generated the shift. The Australian economy also takes several years to recover from a large negative shock even if the attractor remains constant. In Japan, the attractor also shifted in the period following the 1974 oil shock. The extent of the shift compared to Australia was small. There was also a relatively speedier resolution to the 1980s downturn compared to Australia. The Japanese economy now appears to be seeking a new higher attractor although some slowing of the rise is evident in 2001. The US has fluctuated around an attractor unemployment rate of 5.5 to 6 per cent since the early 1960s, although the magnitude of the cycles has been variable. The early 1990s recession, while significant, did not promote a new attractor. So a major difference between Australia and Japan on one hand, and the US on the other, is the sensitivity of the attractor to cyclical events.

Vacancy rate relationships are shown in Figure 2. The 1974-75 disturbances in the unemployment rate attractor in Australia also promoted a shift in the vacancy rate attractor, although in this case the movement was downwards. Supply side analysis interprets the unemployment shift in Figure 1(a) as a decline in labour market efficiency (for example, OECD, 1994). But the shift in Figure 2(a) using the same logic would be interpreted as increasing matching efficiency (Layard, Nickell and Jackman, 1991, hereafter LNJ). Clearly, both states cannot hold. A consistent interpretation can be found in the view that the Australian economy has been demand constrained as a result of a regime shift in government policy in the mid-1970s. The rapid rise in unemployment in 1974 was so large that subsequent economic growth with on-going labour force and productivity growth could not reverse the stockpile of
unemployed (Mitchell, 2001a). Whatever endogenous supply effects that may have occurred in skill atrophy and work attitudes were not causal but reactive.

For Japan and the US, the vacancy rate attractor has not exhibited any notable shifts over the period examined, although the oil price disturbances in 1974 generated negative but temporary impacts on Japan.

In Figure 3, phase relationships for Real GDP annual growth rates for Australia, Japan and the US are shown. For Australia, the attractor growth rate is constant (at around 3.5 per cent per annum) although the severity of the three recessions (mid 1970s, early 1980s and early 1990s) is clearly shown. The pattern is similar for the US with an attractor around 4 per cent per annum, although the depth of the 1980s recession was greater compared to Australia. However, the growth behaviour for Japan is distinct with a clear attractor shift occurring in the 1974 downturn and possibly another shift occurring in the current period. The Japanese economy has, over the sample, moved from being a high growth economy (attractor around 9-10 per cent) to being a modest growth economy (with an attractor around 3-4 per cent per annum).

Figure 4 concentrates on the employment behaviour in Australia differentiating between total, private and public employment over the 1969-2000 period. The data shows that while the decline in total employment growth in the mid-1970s did not reach the negative proportions of the later recessions, the recovery 1975 was very muted compared to subsequent recoveries. This goes a long way to accounting for the rise in unemployment. The sectoral phase relationships extend the story. The private sector employment growth rate experiences large cyclical fluctuations but has oscillated around a relatively fixed attractor. However, a striking result is the negative shift in the public sector employment growth attractor, which began in the second half of the 1980s. At this time private employment growth was unusually strong and if the public sector had maintained their previous labour market behaviour, the unemployment rate would have been driven down closer to full employment thus mopping up a large portion of the stock persistence from the mid-1970s (Mitchell, 2001b). The attractor shift continued apace in the 1990s when for the first time the Australian public sector employment growth became procyclical.

To explore the Australian case further, the phase relationship for labour force growth is shown in Figure 5. From 1962 to 1970 labour force growth averaged 2.6 per cent
per annum. Over the period 1990 to 2001, this had changed to 1.5 per cent per annum. It is apparent that during the 1990s there has been a shift in the labour force growth attractor downwards and the cycle now traces ellipses around a lower rate of growth. Considering all this evidence together we can conclude that the unemployment rate would have been much worse, given the employment growth if the labour force growth attractor had not shifted downwards. The decline in public employment growth has been hidden to some extent by this slowdown in labour force growth.

If we view the shifts in attractor points as endogenous events then they depict non-linear time series behaviour. In a linear AR(1) model, for example, the cyclical ellipses would be clustered around the constant mean. If this non-linearity and asymmetry in unemployment rates is positively skewed (increases are larger and/or swifter than decreases), then public policy should aim not only to shift the attractor to more favourable unemployment rates but should also put in place policies that prevent negative attractor shifts. The presence of non-linearity is examined more formally in the remaining sections of this paper.

3. Non-linearities in time series

3.1 Introduction

Two major alternative hypotheses have arisen to explain the observed inertia-prone behaviour of unemployment rates since the mid-1970s: (a) that the natural rate of unemployment or NAIRU has risen due to structural changes in the labour market; or (b) that the unemployment rate displays hysteresis.

The NAIRU was initially asserted to a constant and cyclically-invariant (LNJ, 1991) despite early challenges from hysteresis theories (Hargreaves-Heap, 1980, Mitchell, 1987). Faced with mounting conceptual criticism and a lack of empirical foundation, the NAIRU theorists progressively moved to espousing time varying steady-states (so-called TV-NAIRU models), although this variation remained cyclically insensitive (for example, Gordon, 1997). This concession spawned a frenetic period of estimation using a range of technical methodologies including state space techniques (Kalman filter); univariate extrapolation methods (filters and smoothers), and spline estimation. However, the NAIRU concept remains on shaky theoretical grounds. Importantly, the original theory underpinning the NAIRU provides no guidance about its evolution although, we would be looking to the evolution of unspecified structural factors to
remain faithful to that theory. In this theoretical void, econometricians have used techniques that allow for a smooth evolution although there is no particular correspondence with any actual economic factors. Some authors have the temerity to assert that a Hodrick-Prescott filter through the actual series captures the TV-NAIRU (for example Boone, 2000 among many). Of course, the Hodrick-Prescott filter merely tracks the underlying trend of the unemployment and follows it down just as surely as it follows it up. The unemployment rate is highly cyclical and the TV-NAIRU proponents are silent on this apparent anomaly – why do the alleged structural factors cycle with the actual rate?

Equally damaging is the evidence from Staiger, Stock and Watson (1997: 46) who use “state-of-the-art” NAIRU estimation and conclude that “these estimates are imprecise.” Some of their models yield confidence intervals of 2.9 percent to 8.3 percent. Chang (1997: 9) concludes, “The range of uncertainty about the location of the NAIRU is often too large to be useful.”

At any rate, the NAIRU approach has little value if unemployment rates behave asymmetrically. Hysteresis models offer more prospects in this regard (Mitchell, 1993). However, the hysteresis hypothesis itself is not without problems. The implication of assuming a fully-hysteretic unemployment rate is that in the absence of drift all unemployment rates represent equilibrium rates. Further, a fully-hysteretic unemployment would be a unit root process (with or without drift) and irrespective of the current level, the probability is equal that the rate will rise or fall (Mitchell, 1987, 1993, 2001a; Skalin and Teräsvirta, 2002). While many researchers have found unit roots in unemployment rates for most countries, the finding has to ultimately be an artefact of the small sample sizes and low power of the existing tests. The unemployment rate is bounded and cannot have an infinite variance. Mitchell (1993) noted this point and distinguished between persistent and hysteretic processes, a previously blurred dichotomy. Even if the NAIRU model represents the true model, persistence means that the effects of shocks have long memories and that short-term macroeconomic policy can be highly effective in attenuating the costs of recession. Mitchell (2001b) among others, finds that unemployment rates in OECD countries are highly persistent following a shock.

In this paper, it is assumed that persistence dominates the time-series behaviour of unemployment rates. Thus the unemployment rate is assumed to be a stationary long-
memory process subject to positive and negative shocks. As Skalin and Teräsvirta (2002: 1) note, the “unemployment rate may … be thought of as remaining on a given level until it is dislodged and pushed to a new level by a shock or a series of shocks.” There is nothing particular about any level and multiple equilibria could exist (Mitchell, 1993; Skalin and Teräsvirta, 2002). This conception suggests that we would require a non-linear depiction to capture the behaviour.

An examination of the behaviour of unemployment rates would also suggest an additional characteristic is important – asymmetry. The concept of asymmetry has been used to refer to the observation that unemployment rates rise rapidly in a downturn and take much longer to decrease. Asymmetry and persistence work together. However, the debate about hysteresis versus persistence does not rest on asymmetry behaviour. They are quite separable. The importance of asymmetric behaviour in series that are also prone to persistence is that the costs of a recession become higher than a series that has equal amplitude and symmetrical cycles.

3.2 Testing asymmetry

Neftci (1984) and DeLong and Summers (1986) revived the empirical literature on asymmetrical behaviour of unemployment rates and showed that the U.S. unemployment rate increases more rapidly than in decreases over the business cycle. A number of nonlinear models have been fitted to unemployment rates since then (see Pfann, 1993 for a survey). More recently, Parker and Rothman (1997), Rothman (1998), and Montgomery, Zarnowitz, Tsay and Tiao (1998) have developed nonlinear models to forecast unemployment because they have found them superior to linear representations. There has also been substantial development of the threshold autoregressive (TAR) class of models (see Hansen, 1997) and the smooth transition autoregressive (STAR) class of models and their derivatives (see Skalin and Teräsvirta, 2002). There is now a strong body of literature supporting the conclusion that many unemployment rates display non-linear behaviour.

In this section, we seek to test for cyclical asymmetries in unemployment rates for Australia, Japan and the US. Sichel (1993) outlines two concepts of business cycle asymmetry: (a) deepness, and (b) steepness. In the business cycle context, deepness occurs when “troughs are deeper than the peaks are tall”, whereas steepness occurs “when contractions are steeper than expansions” (Sichel, 1993: 225). A symmetric
cycle indicates the absence of both sources of asymmetry. The three concepts are shown for a trendless procyclical time series in Figure 6. McQueen and Thorley (1993) add sharpness or turning point asymmetry to the taxonomy. Where sharpness is present, troughs are V-shaped and peaks are U-shaped. We do not consider sharpness asymmetry in this paper (see Mitchell, 2001a for turning point analysis of the Australian labour market). The formal definitions of the respective concepts of cyclical asymmetry are shown in Table 1 (see also Clements and Krolzig, 2002). The definitions are for a pro-cyclical time series. Given unemployment is a counter-cyclical time series, its asymmetry will mirror the behaviour of the pro-cyclical data.

3.3 Deepness and steepness

This section employs the skewness test developed by Sichel (1993). We begin with a time series:

\[ y_t = \tau_t + c_t + \xi_t \]

where \( \tau_t \) is a nonstationary trend component, \( c_t \) is a stationary cyclical component and \( \xi_t \) is an irregular component assumed to be white noise. Sichel (1993) focused on the asymmetry in the cyclical component (assuming that the trend component was usually asymmetric in that it was almost always increasing).

Sichel (1993: 227-228) argues that “If a time series exhibits deepness, then it should exhibit negative skewness relative to mean or trend; that it is should have fewer observations below its mean or trend than above, but the average deviation of observations below the mean or trend should exceed the average deviation of observations above ...[and if]... a time series exhibits steepness, then its first differences should exhibit negative skewness. That is, the sharp decreases in the series should be larger, but less frequent, than the moderate increases in the series.”

The coefficient of skewness (for deepness) is:

\[ D(c) = \frac{T^{-1} \sum (c_i - \mu_c)^3}{\sigma(c)^3} \]

where \( \mu_c \) is the mean and \( \sigma(c) \) is the standard deviation of \( c \), and \( T \) is the number of observations. \( D(c) = 0 \) for a symmetric distribution. Positive (negative) skewness means the distribution has a long right (left) tail.
To estimate this coefficient and generate an asymptotic standard error for $D(c)$ we construct the following variable using the Newey and West (1987) procedure:

$$z_t = \left( c_t - \mu_c \right)^3 / \sigma(c)^3$$

This variable is regressed on a constant and a heteroskedasticity-autocorrelation consistent Newey-West standard error computed. Sichel (1993) shows that the estimate of the constant is identical to $D(c)$ with a consistent standard error. The $t$-statistic for the constant is asymptotically normal and thus the significance of $D(c)$ is easily tested using the normal distribution.

The steepness test statistic seeks to determine whether the change in $c_t$ are asymmetric around their mean and is computed using the coefficient for skewness for the first-difference $\Delta c_t$:

$$ST(\Delta c_t) = \left[ T^{-1} \sum \left( \Delta c_t - \mu_{\Delta c} \right)^3 \right] / \sigma(\Delta c)^3$$

where $\mu_{\Delta c}$ is the mean and $\sigma(\Delta c)$ is the standard deviation of $\Delta c$. The procedure to complete the test is identical to the deepness test (see also DeLong and Summers, 1986).

The contentious part of this test lies in the separation of the components in (2). There is no broadly accepted best way to filter out the cyclical components of a time-series. Sichel (1993: 228) outlines three requirements a detrending filter has to satisfy to ensure it does not itself introduce asymmetry and the test statistics, $D(c)$ and $ST(\Delta c)$ have standard distributions: (a) “the detrending filter has a linear representation” – a linear filter cannot induce spurious asymmetry, (b) “the filter must induce stationarity” – this ensures the asymmetry tests have standard distributions, and (c) “the detrending filter used for each test must extract the component appropriate for that asymmetry test” – that is, a deepness filter must extract $c_t$ and steepness filter must extract in $\Delta c_t$.

Several filters satisfy condition (a) but violate one or both of (b) and (c). Simple linear detrending may not produce (b) in the case of integrated processes. Differencing will satisfy (b) irrespective of the source of nonstationarity that may be present but violates (c) because it produce $\Delta c$. This is not the correct component for the deepness test. However, differencing can be used for the steepness tests. The Hodrick-Prescott filter
will satisfy (b), again, even if the process is integrated, and can satisfy (c) in the case of the deepness test. To get the de-trended cyclical component, \( c_t \) a Hodrick-Prescott filter is run through the time series and subtracted from the original. However, this procedure is redundant, given differencing, in the context of the steepness test. Finally, a decomposition of (2) using the structural time series approach (Harvey, 1989), which assumes that the time series can be represented in terms of a set of additive unobservable components that lend themselves to economic interpretation as trend, cycle, seasonal and irregular can generate \( c_t \). Kalman Filter methods are used to generate the decomposition.

As an indication, three estimates of the cyclical component of the Australian unemployment rate derived by the Hodrick-Prescott filter, the Kalman Filter and a simple linear trend are shown in Figure 7. The Hodrick-Prescott and Kalman Filter approaches provide similar results although the well-known property that the Hodrick-Prescott filter tends to amplify business cycle frequencies is evident. The linear decomposition is quite different. The graphs of the de-trended cyclical component for each country are shown in Figure 8 although these charts are based on seasonally adjusted data and the subsequent tests used the same decompositions on seasonally unadjusted data.

### 3.4 Results

The results for Australia, Japan and the U.S. are shown in Table 2. Deepness and steepness statistics for quarterly unemployment rates were computed for each country using both Hodrick-Prescott detrending and non-detrended data. Seasonally unadjusted data was used because it is “not known what may happen to a nonlinear asymmetric series when it is filtered by applying some standard seasonal adjustment procedure” (Skalin and Teräsvirta, 2002: 8). Seasonality was handled using linear dummies. This implicitly assumes that the temporal pattern of seasonality is non-stochastic, which is perhaps justified given the results in Section 5.

All tests of deepness and steepness reject the null hypothesis and so the result is not sensitive to de-trending. The evidence is relatively stronger for Japan overall. The test conclusions concur with the graphical evidence shown in Figure 8 and are consistent with the results of Sichel (1993) and DeLong and Summers (1986).
4. **Current Depth of Regression models of non-linearity**

4.1 **The Current Depth of Regression model**

Mitchell and Muysken (2002b) employ the Current Depth of Recession (CDR) approach, introduced by Beaudry and Koop (1993), to test for non-linearity in unemployment rates for Australia and The Netherlands. The test examines whether there is asymmetry in the response of the unemployment rate to negative and positive shocks, specifically, that negative shocks impact more strongly than positive shocks. They concluded that while the depth of the 1975 recession was similar in both countries, it impacted for longer in Australia. The Dutch economy seemed to absorb the negative shock better. History tells us that the Australian labour market has never generated enough employment growth since 1975 to keep pace of the labour force and productivity growth and the stock-pile of unemployed that occurred in that period.

Beaudry and Koop (1993) constructed the CDR variable as the difference between the previous maximum value for GNP minus the current value. We follow Mitchell and Muysken (2002b) who construct the CDR for the unemployment rate as (see also Parker and Rothman, 1998):

\[
CDR_t = \min \{U_{t-1} \}_{t=0,\ldots,5} - U_t
\]

The difference between this approach and the Beaudry and Koop (1993) construction reflects the fact that the unemployment rate is a cyclical variable with no evidence of a strong trend. Accordingly, the minimum is defined as the lowest value for the last 6 quarters (a local minimum). The CDR is then the difference between this value and the actual unemployment rate. When the unemployment rate is above this local minimum, the CDR variable is negative and measures the depth of recession.

The unemployment rates and the CDR variables for Australia, Japan and the US are shown in Figure 9. The comparison shows that all labour markets went through similarly dated recessions although the magnitudes in each were substantially different. Japan’s recent troubles are clearly at variance to the experience of Australia and the US. The bi-lateral comparisons (Australia and Japan; Australia and the USA) that are shown in Figure 10 are revealing. The Japanese economy experienced smaller recessions (using the CDR as the indicator) in the early 1980s and the early 1990s.
4.2 Results

We then estimated the CDR effect in an autoregressive model (see Parker and Rothman, 1998). The results for the preferred equations for Australia, Japan and the United States are presented in Table 3. In each case, the lag order was determined by reference to AIC and the CDR model provides a reduction in residual variance compared to the tested-down linear AR model. The results confirm the presence of the CDR effect in each country with the US exhibiting the strongest effect (as measured by the coefficient magnitude), followed by Australia. The significant negative CDR coefficient indicates that the unemployment rate increases quickly in recessions, but declines relatively slowly during expansions. It is interesting that the regressions imply that the US exhibits less inertia overall than Australia and Japan (as indicated by the sum of the AR coefficients). The results confirm that a negative shock imposes higher costs on the labour market being more persistent than the opposite shock. The results also show that the impact of the shock depends on the current state of the labour market. A depressed labour market that is then confronted with restrictive macroeconomic policy will be driven deeper into a sustained period of high unemployment.

The forecasting accuracy of the CDR model is compared to that of the relevant linear AR model in Table 4. On all indicators the CDR model is superior for each country. In Table 5, the comparison between the linear and CDR models by AIC and variance size is shown and again confirms the superiority of the CDR model.

The results in general suggest that if one wishes to understand the evolution of unemployment rates in the countries examined then it is important to take into account the non-linear behaviour that is evident. The typical NAIRU models fail in this regard.

5. Smooth transition autoregressive models

5.1 The STAR class of Models

The literature on non-linear econometric modelling has expanded rapidly in recent years (van Dijk, Teräsvirta and Franses, 2001 provide a good survey). A popular class of model is the smooth transition autoregressive (STAR) model that has been used successfully to capture regime shifts associated business cycle asymmetry, that is, the differential impact of expansions and recessions (see Teräsvirta, 1994, 1998). The
STAR model class allows us to distinguish behaviour in the two periods of activity and the timing of the transition between the regimes. In that sense, they are useful vehicles for the research agenda pursued in the paper.

Following Teräsvirta (1994), the $p$-order STAR model is written as:

$$y_t = \delta_1 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \left( \delta_2 + \sum_{i=1}^{p} \alpha_{2i} y_{t-i} \right) G(z_t; \gamma, c) + \epsilon_t$$

where $\epsilon_t \sim IID(0, \sigma^2)$. The model can be generalised to include exogenous regressors in deterministic trends and seasonal variables. The framework allows for errors with asymmetric conditional variances (for example, the STAR-GARCH class).

The specification of the transition function $G(z_t; \gamma, c)$ determines the behaviour of the time series. It is assumed to be a continuous function bounded between 0 and 1. To make the analysis tractable a specific form for the transition function is required. In this study, it is assumed that the transition variable $z_t$ is a lagged endogenous variable ($y_{t-d}$ for $d > 0$) although the model can accommodate exogenous transition variables with a variety of functional forms.

How do we interpret the STAR model? We may consider it in the class of regime-switching models with two regimes that occur when the transition function takes its extreme values of 0 and 1. The transition between these regimes is smooth. We may also consider a continuum of regimes that are determined by the range of values the transition function can take between 0 and 1.

Thus if $G(.) = 0$, the model (6) reduces to a linear AR($p$) with parameters $\delta_1, \alpha_{1i}$, where $i = 1$ to $p$. If $G(.) = 1$ the extra AR($p$) parameters $\delta_2, \alpha_{2i}$ where $i = 1$ to $p$ are explanatory. For values of $G(.)$ between 0 and 1, the model can be interpreted as “the weighted sum of these two individually linear AR models” (Franses, 1998: 176). So:

$$y_t = \left( \delta_1 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} \right) (1 - G(z_t; \gamma, c)) + \left( \delta_2 + \sum_{i=1}^{p} \alpha_{2i} y_{t-i} \right) G(z_t; \gamma, c) + \epsilon_t$$

The transition function $G(z_t; \gamma, c)$ can be represented by a number of different forms with two popular models being the exponential STAR (ESTAR) model) and the first-order logistic STAR (LSTAR) model. We use the LSTAR approach in this section.
Thus the transition function is written as:

\[ G(y_{t-d}) = \left[ 1 + \exp(-\gamma(y_{t-d} - c)) \right]^{-1}, \quad \gamma > 0 \]

The two logistic parameters, \( \gamma \) and \( c \), describe the speed and smoothness of the transition between regimes. The parameter \( c \) is termed the threshold parameter between the two regimes. When \( y_{t-d} = c \), \( G(z_t; \gamma, c) = 0.5 \). As the transition variable changes relative to \( c \) the logistic function changes monotonically from 0 to 1. So the LSTAR model switches from one regime to another as the transition variable shifts relative to \( c \). The parameter \( \gamma \) estimates the steepness of the transition between regimes. The larger is \( \gamma \) the steeper is the transition. For very large values of \( \gamma \) the transition function switches instantaneously between the regimes. The characteristics of a stylised set of LSTAR transition functions are shown for different values of the logistic parameters in Figure 11.

Granger and Teräsvirta (1993) outline the following specific-to-general methodology for non-linear modelling:

1. Estimate a linear AR(\( p \)) model using AIC to determine parsimony;
2. Test the null hypothesis of linearity against the non-linear (STAR) alternative;
3. If the null is rejected, choose the transition variable and the functional form for \( G(z_t; \gamma, c) \);
4. Estimate the STAR model and conduct appropriate diagnostic tests.

### 5.2 Results

The experimental analysis suggested that there was a lot of noise in the quarterly changes in the unemployment rate and so the first difference may not “represent variations in the unemployment cycle, and smoothing in the form of a longer difference becomes necessary” (Skalin and Teräsvirta, 2001: 9). The annual difference of the unemployment rate lagged one period proved to be the best variable (using AIC criteria) to smooth the transition. The initial model was tested using seasonally unadjusted data over a shorter sample period due to data availability (1980:1 to 2001:4). Seasonal variables were allowed to enter the non-linear part of the model but were statistically insignificant. The results were compared with a version derived from seasonally adjusted data over the same sample and the outcomes were
close. To gain the benefits from another 20 years of observations, the final model examined here is based on the seasonally adjusted data. The first-difference of the unemployment rate is used on the assumption that it is globally stationary but may be nonlinear and locally nonstationary (see van Dijk, Teräsvirta and Franses, 2001:41).

The tests for linearity against STAR-type non-linearity proposed by Teräsvirta (1994) were performed prior to estimation. The tests generally reject the null of linearity at the 5 per cent level. The estimation of the LSTAR parameters was achieved using the Broyden, Fletcher, Goldfarb, Shanno (BFGS) method within an Ox Pack written in the Ox object-oriented matrix programming language. My own program was initially motivated by an earlier translation of Teräsvirta’s GAUSS code by Ivar Pettersen, which is available at http://www.sv.ntnu.no/iso/Ivar.Pettersen. Earlier attempts were made to program E-Views 4.0 to estimate the non-linear parameters but the software was unable to generate satisfactory standard errors. The final model had the highest $R^2$ of the non-linear specifications tried after starting with a general model (8 lags of the dependent variables) and various transition variable lags.

The final estimated LSTAR model for the period 1961(2) to 2001(4) is:

\[
\begin{align*}
(9) \Delta_4 y_t &= -0.26 - 0.26 \Delta_1 y_{t-4} + 0.72 D74 + 1.38 D82 + \left(0.87 - 0.43 \Delta_1 y_{t-4}\right) \tilde{G}(\Delta_4 y_{t-1}) + \tilde{\epsilon}_t \\
R^2 &= 0.429 (0.417) \quad \text{s.e.} = 0.274 (0.276) \quad \text{AIC} = -2.54 (0.308)
\end{align*}
\]

with the estimated transition function as

\[
(9a) \quad \tilde{G}(\Delta_4 y_{t-1}) = \left[1 + \exp(-1.700(\Delta_4 y_{t-1} - 0.525))\right]^{-1}
\]

where the $t$-statistics are shown in the parentheses below the coefficient estimates. The diagnostic statistics in brackets are for an AR(4) linear model, which satisfied serial correlation tests. $D74$ and $D82$ are impulse dummies for the 3rd and 4th quarters in 1974 and the 4th quarter in 1982, respectively. They were used to control for severe outliers in those quarters. The estimation of the logistic parameters was not sensitive to their inclusion. RESET misspecification tests were conducted for 2nd and 3rd order terms with p-values of 0.3796 and 0.4876, respectively indicating that we cannot reject the null of no misspecification. The LM tests for up to 4th order serial correlation were also encouraging.
The actual and fitted values and linear and non-linear residuals are shown in Figure 12. It can be seen that the nonlinear model has smaller residuals when the unemployment rate is rising steeply, and (Skalin and Teräsvirta, 2001: 12) consider this a “major contribution of the nonlinear model”.

The time series plot of the transition function is shown in Figure 13(a). The cross plot of the transition function and the transition variable is shown in Figure 13(b) and the relationship between the $c$ (0.525) parameter and the transition variable is shown in Figure 13(c). The mean of the transition variable $\Delta_4y_{t-1}$ over the sample period is 0.1237 and the standard deviation of 1.03. From Equation (9a), it is clear that if $\Delta_4y_{t-1}$ is greater than 0.525, the estimated LSTAR transition function $G(\Delta_4y_{t-1}; \gamma, c)$ approaches unity, which we interpret as a recession. The economy moves between recession and expansion relatively smoothly.

Using the benchmark of $\Delta_4y_{t-1} > 0.525$ is a recession quarter, the LSTAR model provides an alternative method of computing turning points (peaks and troughs) in the business cycle (although we no that the unemployment rate leads at peaks and lags at troughs measured by real GDP cycles). The striking point to emerge from these computations shown in Table 5 is the extent to which the unemployment rate makes the transition into recession even when the real GDP growth rate does not deteriorate sufficiently to warrant the term recession being used. Australia’s labour market has been depressed consistently over the 1970-2001 period.

6. Conclusion

The research in this paper strongly suggests that unemployment rates in Australian, Japan and the US exhibit non-linearity although the degree of persistence following a shock is lower for the US. The econometric analysis underpins the graphical evidence displayed in Section 2. The relatively poorer unemployment outcomes in Australia compared to Japan and the US is related to the failure of the Australian economy to maintain a stable unemployment and vacancy attractor. In particular, the data suggests that in the Australian case, the economy reacts more badly to recession. The conduct of macroeconomic policy in Japan and the US has also been less driven by the NAIRU “fight inflation first” rhetoric that has dominated the Australian policy debate (Mitchell, 2001a). The non-linearity suggests the two phases of the business cycle are very important for demand side policy. First, there should be a priority placed on
maintaining levels of demand such that business cycle downturns are muted. Second, in the event of a private sector spending collapse, government must ensure that fiscal (and monetary) policy returns the economy to a higher growth path quickly to avoid any negative shifts in attractors.

For Australia, the phase diagram analysis suggests that to restore full employment, the economy needs a major positive employment shock of a sufficient magnitude to shift the current attractor point downwards (Ormerod, 1994: 161). It is almost definite from the earlier analysis and related empirical work reported in Mitchell and Muysken (2001a) that this shock has to be aggregate and focused on employment, given the lack of a close correspondence between real GDP growth and reductions in unemployment. The thrust of governments within the OECD have been on supply initiatives to improve the efficiency of the labour market matching process and to withdraw alternative sources of assistance which might erode incentives to work (OECD, 1994). Layard (1997) has recently cast doubt on the supply-side labour market policies that he initially promoted in LNJ (1991) and which were so zealously taken up by the OECD and governments around the world. Layard (1997: 202) concludes that “If we seriously want a big cut in unemployment, we should focus sharply on those policies which stand a good chance of having a really big effect. It is not true that all policies which are good in general are good for unemployment. There are in fact very few policies where the evidence points to any large unambiguous effect on unemployment and … some widely advocated policies for which there is little clear evidence.” He included changes to “social security taxes”, changes to “job protection rules”, “productivity improvements”, and “decentralizing wage bargaining” as “policies whose effects are difficult to forecast”. For example, Layard (1997: 192) argues that further cuts in the duration of benefits would only increase employment at the costs of the creation of an underclass with an “ever-widening inequality of wages.” He now prefers government job creation, which would allow people to reacquire “work habits … to prove their working capacity … [and to restore] … them to the universe of employable people. This is an investment in Europe’s human capital.” (Layard, 1997: 192)

This paper suggests that there is a clear need for direct job creation to shift the attractors for unemployment and vacancies in the direction necessary to secure full
employment. The responsibility for this policy shift lies squarely in the lap of the Federal government.
References


Figure 1 Unemployment rate relationships, Australia, Japan and the US, 1961-2001.

(a) Australia 1961-2001

(b) Japan 1961-2001

(c) United States 1961-2001

Source: ABS, The Labour Force, 6203.0, OCED, Main Economic Indicators, various years.
Figure 2 Vacancy rate relationships, Australia, Japan and the US, various years.

(a) Australia 1968-2001

(b) Japan 1960-2000

(c) United States 1961-2000

Source: ABS AUSSTATS, OCED, Main Economic Indicators, various years.
Figure 3 Real GDP growth rate relationships, Australia, Japan and the US.

(a) Australia 1962-2001

(b) Japan 1962-1999

(c) United States 1962-2001

Source: ABS AUSSTATS, OCED, Main Economic Indicators, various years.
Figure 4 Employment growth rate relationships, Total, Private, and Public, Australia.

(a) Total employment 1969-2000

(b) Private sector employment 1969-2000

(c) Public sector employment 1969-2000

Source: ABS AUSSTATS.
Figure 5 Labour force growth rate relationship, Australia, 1962-2001

Source: ABS *The Labour Force*, 6203.0. The growth rate is the annual percentage change.
Figure 6 Symmetric and asymmetric cycles for a trendless pro-cyclical time series

These diagrams are based on Sichel (1993: 226, Figure 1). In the case of a counter-cyclical time series like the unemployment rate, steepness manifests as rapid and large increases in the unemployment rate and slow declines as the economy improves. Deepness manifests as very high and sharp peaks occurring coincident with the trough of the cyclical variable.
Table 1 Definitions of steepness, deepness and sharpness

<table>
<thead>
<tr>
<th>Concept</th>
<th>Description</th>
<th>Condition</th>
<th>Skewness Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deepness</td>
<td>“Troughs are deeper than peaks are tall” (Sichel, 1993: 225)</td>
<td>The process ( { x_t } ) is said to be non-deep (non-tall) ( iff ) ( x_t ) is not skewed.</td>
<td>Negative skewness indicates deep contractions: ( E \left( c_t - \mu_c \right)^3 &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>“… it should have fewer observations below its mean or trend than above, but the average deviation of observations below the mean or trend should exceed the average deviation of observations above” (Sichel, 1993: 227)</td>
<td>Thus: ( E \left( c_t - \mu_c \right)^3 = 0 )</td>
<td>Positive skewness indicates deep expansions: ( E \left( c_t - \mu_c \right)^3 &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>Relates to relative levels around the mean or trend.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steepness</td>
<td>“Contractions are steeper than expansions” (Sichel, 1993: 225)</td>
<td>The process ( { x_t } ) is said to be non-steep ( iff ) ( \Delta x_t ) is not skewed.</td>
<td>Steep contractions: ( E \left( \Delta c_t \right)^3 &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td>“Sharp decreases in the series should be larger, but less frequent, than the more moderate increases” (Sichel, 1993: 228)</td>
<td>Thus: ( E \left( \Delta c_t \right)^3 = 0 )</td>
<td>Steep expansions: ( E \left( \Delta c_t \right)^3 &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>Relates to relative slopes or rates of change.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sharpness</td>
<td>Troughs are sharp and peaks more rounded (McQueen and Thorley, 1993)</td>
<td>The process ( { x_t } ) is said to be non-sharp ( iff ) the transition probabilities to and from the two outer regimes are identical. In a two regime model, for example, this requires ( p_{12} = p_{21} ).</td>
<td>n.a.</td>
</tr>
<tr>
<td></td>
<td>The switch from contraction to high growth is more likely than a switch from high growth to contraction.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7 Cyclical decompositions of the Australian unemployment rate

- Linear
- Hodrick-Prescott
- Kalman Filter
Figure 8 Unemployment rates, HP and detrended cyclical component

(a) Australia - 1959(3) to 2001(4)

(b) Japan - 1960(1) to 2001(4)

(c) USA - 1960(1) to 2001(4)

Source: ABS 6203.0, OECD, Main Economic Indicators
Table 2 Deepness and steepness in unemployment rates, Australia, Japan and the US

<table>
<thead>
<tr>
<th>Deepness</th>
<th>Filter</th>
<th>$D(c)$</th>
<th>Asymptotic Std. Err.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>HP</td>
<td>2.25</td>
<td>0.98</td>
<td>0.011</td>
</tr>
<tr>
<td>(1959:3 to 2001:4)</td>
<td>NT</td>
<td>1.82</td>
<td>0.82</td>
<td>0.013</td>
</tr>
<tr>
<td>Japan</td>
<td>HP</td>
<td>4.14</td>
<td>0.85</td>
<td>0.000</td>
</tr>
<tr>
<td>(1960:1 to 2001:4)</td>
<td>NT</td>
<td>1.28</td>
<td>0.75</td>
<td>0.044</td>
</tr>
<tr>
<td>USA</td>
<td>HP</td>
<td>3.35</td>
<td>1.39</td>
<td>0.008</td>
</tr>
<tr>
<td>(1960:1 to 2001:4)</td>
<td>NT</td>
<td>1.75</td>
<td>1.07</td>
<td>0.051</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Steepness</th>
<th>Filter</th>
<th>$ST(\Delta c)$</th>
<th>Asymptotic Std. Err.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>HP</td>
<td>3.63</td>
<td>0.89</td>
<td>0.000</td>
</tr>
<tr>
<td>(1961:1 to 2001:4)</td>
<td>NT</td>
<td>3.61</td>
<td>1.05</td>
<td>0.000</td>
</tr>
<tr>
<td>Japan</td>
<td>HP</td>
<td>4.11</td>
<td>0.64</td>
<td>0.000</td>
</tr>
<tr>
<td>(1961:1 to 2001:4)</td>
<td>NT</td>
<td>3.83</td>
<td>0.56</td>
<td>0.000</td>
</tr>
<tr>
<td>USA</td>
<td>HP</td>
<td>5.38</td>
<td>2.04</td>
<td>0.004</td>
</tr>
<tr>
<td>(1961:1 to 2001:4)</td>
<td>NT</td>
<td>5.33</td>
<td>2.12</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Filter refers to the method of deriving the cyclical component. HP is Hodrick-Prescott. NT is no trend removed. The p-values are taken from the normal distribution based on a one-tail test.
Figure 9 Unemployment rates and CDR for Australia, Japan and the U.S.A.

(a) Australia 1960(4) to 2001(4)

(b) Japan 1960(4) to 2001(4)

(c) USA 1960(4) to 2001(4)

Source: ABS, The Labour Force, 6203.0 and OECD, Main Economic Indicators.
Figure 10 CDR variable for Australia, Japan and the USA

(a) Australia and Japan CRD Comparison

(b) Australia and USA CRD Comparison

Source: see Figure 9
### Table 3 CDR regression results for Australia, Japan and the United States

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.096</td>
<td>-0.024</td>
<td>0.472</td>
</tr>
<tr>
<td></td>
<td>(1.64)</td>
<td>(1.32)</td>
<td>(2.39)</td>
</tr>
<tr>
<td>UR(-1)</td>
<td>0.976</td>
<td>1.015</td>
<td>0.379</td>
</tr>
<tr>
<td></td>
<td>(100.6)</td>
<td>(118.2)</td>
<td>(2.97)</td>
</tr>
<tr>
<td>UR(-2)</td>
<td></td>
<td></td>
<td>0.264</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.92)</td>
</tr>
<tr>
<td>UR(-3)</td>
<td></td>
<td></td>
<td>-0.265</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.89)</td>
</tr>
<tr>
<td>UR(-4)</td>
<td></td>
<td></td>
<td>0.497</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.89)</td>
</tr>
<tr>
<td>CDR</td>
<td>-0.118</td>
<td>-0.075</td>
<td>-0.413</td>
</tr>
<tr>
<td></td>
<td>(3.76)</td>
<td>(1.94)</td>
<td>(3.69)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.344</td>
<td>.093</td>
<td>.588</td>
</tr>
<tr>
<td>s.e. % mean</td>
<td>6.14</td>
<td>4.13</td>
<td>9.89</td>
</tr>
<tr>
<td>Mean of UR</td>
<td>5.60</td>
<td>2.27</td>
<td>5.94</td>
</tr>
<tr>
<td>S.D of UR</td>
<td>2.91</td>
<td>1.02</td>
<td>1.56</td>
</tr>
</tbody>
</table>

$t$-statistics in parentheses. The s.e. % statistic is the standard error of the regression expressed as a percentage of the mean of the dependent variable.
Table 4 Forecasting comparisons for CDR and linear models

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th></th>
<th>Japan</th>
<th></th>
<th>USA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CDR</td>
<td>Linear</td>
<td>CDR</td>
<td>Linear</td>
<td>CDR</td>
<td>Linear</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.487</td>
<td>1.3925</td>
<td>0.8243</td>
<td>0.9279</td>
<td>0.7047</td>
<td>1.1620</td>
</tr>
<tr>
<td>MAE</td>
<td>0.3926</td>
<td>1.1815</td>
<td>0.7268</td>
<td>0.8042</td>
<td>0.6052</td>
<td>1.0682</td>
</tr>
<tr>
<td>MA%E</td>
<td>5.8184</td>
<td>17.565</td>
<td>15.588</td>
<td>17.106</td>
<td>14.34</td>
<td>25.09</td>
</tr>
<tr>
<td>Theil IC</td>
<td>0.0337</td>
<td>0.0893</td>
<td>0.1017</td>
<td>0.1156</td>
<td>0.0736</td>
<td>0.1157</td>
</tr>
</tbody>
</table>

CDR is the forecast from the CDR model and Linear refers to the forecast with the CDR variable absent. RMSE is the root mean squared error, MAE is the mean absolute error, MA%E is the mean absolute percentage error and Theil IC is Theil’s inequality coefficient. The estimation sample was 1961:1 to 1996:4 and the forecasting horizon was 1997:1 to 2001:4.

Table 5 Comparison between linear and CDR models by variance and AIC

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th></th>
<th>Japan</th>
<th></th>
<th>USA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>CDR</td>
<td>%</td>
<td>Linear</td>
<td>CDR</td>
<td>%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.1242</td>
<td>0.1172</td>
<td>5.64</td>
<td>0.0089</td>
<td>0.0088</td>
<td>1.86</td>
</tr>
<tr>
<td>AIC</td>
<td>0.7641</td>
<td>0.7124</td>
<td>-1.8673</td>
<td>-1.8798</td>
<td>1.8016</td>
<td>1.8103</td>
</tr>
</tbody>
</table>

The % column refers to the percentage improvement in variance as a consequence of adding the CDR variable to the linear autoregressive model. Variance is the variance of the regression and AIC is the Akaike Information Criteria.
Figure 11 Transition function behaviour in LSTAR models

(a) Varying the steepness parameter with constant threshold parameter

(b) Varying the threshold parameter with constant steepness parameter
Figure 12 Actual and Fitted values and residuals from LSTAR model

(a) Actual (solid) and fitted (---) values

(a) Linear and non-linear residuals

The linear residuals were derived from the best fitting linear AR model.
Figure 13 Estimated transition function and relation to transition variable

(a) Estimated $LSTAR$ transition function for Australian unemployment rate

(b) Scatter plot of the $G(\Delta_{t-1}y_t; \gamma, c)$ against $\Delta_{t-1}y_t$

(c) Transition variable $\Delta_{t-1}y_t$ and transition parameter $c$
Table 6 Business cycle peaks and troughs using LSTAR transitions and NBER dating methods based on real GDP, Australia.

<table>
<thead>
<tr>
<th>LSTAR transitions basis</th>
<th>Real GDP Growth basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>Trough</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>September 1974</td>
<td>March 1976</td>
</tr>
<tr>
<td>June 1977</td>
<td>December 1978</td>
</tr>
<tr>
<td>September 2001</td>
<td>September 2000</td>
</tr>
</tbody>
</table>

From the LSTAR model, a peak is the quarter preceding the quarter where $\Delta_y_{t-1} > 0.525$ and the trough is the last quarter where this condition holds in the sequence. The NBER dating analysis was published in Mitchell (2001c) and is based on real GDP trends.